1) An ice cream stand has five different flavors – vanilla, mint, chocolate, strawberry, and pistachio. A group of children come to the stand and each buys a double scoop cone with two different flavors of ice cream. If none of the children choose the same combination of flavors (vanilla on top of chocolate is the same as chocolate on top of vanilla), and every different combination of flavors is chosen, how many children are there?

2) A restaurant offers five-course meals consisting of a soup, salad, appetizer, entree, and dessert. The restaurant has 3 soups, 2 salads, 5 appetizers, 3 entrées, and 6 desserts. How many unique five-course meals could be created?

3) Jared, Mary, Luke, and Terry line up single file to board a roller coaster. Terry refuses to be first in line. In how many ways is it possible for the teens to line up?

4) On a piano, a chord is played by simultaneously pressing down 3 different notes out of the 12 total notes in an octave. How many different chords are possible?

5) A club has 10 members and plans to hold elections for President, Vice-President, Secretary and Treasurer. Each person can only hold one office. How many different ways can these positions be filled?

6) Mr. and Mrs. Smith wanted to have a bit of fun naming their first child, so they decided to choose the letters for their child's name randomly. The only requirements they had for the name were that it is three letters long, that the first and last letters are vowels (a, e, i, o, or u), and that the middle letter is a consonant. How many possibilities are there for that that poor child’s name?

7) In how many different ways can the letters in the word “CENSUS” be uniquely arranged?

8) Five boys and four girls go to a movie and sit in the same row. If no boy sits next to another boy, and no girl sits next to another girl, how many ways can the 9 kids arrange themselves?

BONUS PROBLEMS

9) On Saturday Megan likes to go to Sandwich Heaven. Megan chooses the bread, filling, and topping for her sandwich. She can choose between wheat, rye, or white bread. She can have chicken salad, turkey, or ham for the filling. She can also have lettuce, tomato, or sprouts on top. How many different sandwiches can Megan choose from if she always chooses a filling and a topping with the bread?

10) Sitting in front of you is a locked safe. If you know that the five digits in the code to unlock the safe are 2, 3, 3, 8, and 9, how many unique codes are possible?

11) The lock on your diary allows you to set a three-digit combination. To make it harder on the rest of your family though, you decide that each digit must be different. How many possible combinations do you have to choose from?

12) Four friends, Amber, Becky, Callie, and Darcy went to a party. If Callie was the third to arrive, how many different orders could the other three friends have arrived in?

Solutions

Note: There are many acceptable strategies to solving each problem. This sheet shows just one strategy.

1) There are 5 choices for the bottom flavor. For each of those choices, there are 4 remaining choices for the top flavor (since the two flavors must be different). So, the total number of permutations is 5 x 4 = 20. However, the question states that order doesn't matter … vanilla on top of chocolate is considered the same as chocolate on top of vanilla. In our calculation above, we counted each combination twice, so we must divide by 2 to get the correct number of combinations. 20 ÷ 2 = 10.

   $$\frac{5 \times 4}{2} = 10$$

   Answer: 10

2) For each of the 3 soup choices, there are 2 salad choices. So there are 3 x 2 = 6 total soup/salad combinations. For each of those, there are 5 appetizer choices. So there are 6 x 5 = 30 soup/salad/appetizer choices. And so on …

   3 x 2 x 5 x 3 x 6 = 540

   Answer: 540

3) There are 3 choices for the 1st position (since Terry refuses). Once we've found someone for the 1st position, there are 3 remaining choices for the 2nd position, then 2 remaining choices for the 3rd position, and just one remaining choice for the 4th position. So …

   3 x 3 x 2 x 1 = 18

   Answer: 18

4) For the first note, there are 12 choices. For the second note, there are 11 remaining choices. And for the third note, there are 10 remaining choices. So the total number of permutations is 12 x 11 x 10 = 1320. However, because all 3 notes in the chord are played simultaneously, order doesn't matter. In other words, chord C-E-G is the same as E-G-C. In fact, with any given 3-note chord, there are 6 different ways (3 x 2 x 1) we could have sequenced the notes (e.g., CEG, CGE, EGC, ECG, GCE, GEC) which shouldn't have been counted separately. So we must divide our original answer by 6:

   $$\frac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220$$

   Answer: 220

5) There are 10 choices for President, 9 remaining choices for VP, 8 remaining choices for Secretary, and 7 remaining choices for Treasurer.

$$10 \times 9 \times 8 \times 7 = 5040$$

Answer: 5040

6) There are 5 vowel choices for the first letter, 21 consonant choices for the second letter, and 5 vowel choices for the third letter. (There is no restriction that the first and third letters must be different.) So …

$$5 \times 21 \times 5 = 525$$

Answer: 525

7) There are 6 letters to choose from for the first position, 5 remaining letters for the second position, etc. So there are 6 x 5 x 4 x 3 x 2 x 1 = 720 total permutations. However, there are two letter S's in our pool of letters, and they are indistiguishable from each other. So, which letter 'S' we pull first shouldn't matter. We must not count a given spelling twice (e.g., S_1ENUCS_2 vs. S_2ENUCS_1) just because we pulled one 'S' versus the other. But in our above calculation we have essentially counted every word twice. So we must divide by 2 to get our final answer:

$$\frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2} = 360$$

Answer: 360

8) Since boys don't sit next to other boys, and girls don't sit next to other girls, the only possibly gender arrangement is: boy – girl – boy – girl – boy – girl – boy – girl – boy

So we just need to figure out how many possible arrangements there are of the 5 boys, and separately figure out the number of possible arrangements of the 4 girls. For the boys, there are 5 choices for the first position, 4 choices for the second position, etc. So there are:

$$5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ arrangements for the boys}$$

For the girls, there are:

$$4 \times 3 \times 2 \times 1 = 24 \text{ arrangements for the girls}$$

For each of the 120 boy arrangements, you can choose any of the 24 girl arrangements. So, the total possible arrangements for all 9 kids are:

$$120 \times 24 = 2880$$

Answer: 2880

9) # breads × # fillings × # toppings

   3 × 3 × 3

   27

   Answer: 27 sandwiches

10) There are 5! total ways that the five digits can be sequenced. But since there are two 3's, the order of the 3's don't matter, so we have to divide by 2!

$$nCk = \frac{nPk}{k!} = \frac{5!}{2!} = 60$$

   Answer: 60

11) There are 10 possibilities for the first digit, then 9 remaining possibilities for the second digit, and finally 8 remaining possibilities for the third digit. So, 10 × 9 × 8 = 720. Alternatively,

$$nPk = \frac{10!}{(10-3)!} = 720$$

   Answer: 720

12) There are three possibilities for position 1, then two remaining possibilities for position 2. Position 3 has already been decided for us (Callie), and there is just one remaining possibility for position 4.

   3 × 2 × 1 × 1 = 6

   Answer: 6